CBSE Board

Class IX Mathematics

Sample Paper 7

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Use of calculator is **not** permitted.

Section A

(Questions 1 to 6 carry 1 mark each)

1. If $\left(\sqrt{5}+\sqrt{6}\right)^2 = a + b\sqrt{30}$, then find the values of a and b.

OR

Examine whether $\left(\sqrt{3}+2\right)^2$ is rational or irrational?

2. What is the value of a polynomial f (x) = $8x^2 - 3x + 7$ at x = –1?

OR

Factorise : $x^3 + 27$

3. In quadrilateral PQRS, PQ = QR and RS = SP, then what you can say about the quadrilateral?

4. Comment on the graph of the linear equation 3x = 4.

5. Find the Number of classes, if the class size is 15 and maximum and minimum values are 159 and 69 respectively.

6. The sides of the given triangle are 6 cm, 8 cm and 10 cm, then what is the value of semi-perimeter of a triangle?

(Questions 7 to 12 carry 2 marks each)

7. Evaluate: $\sqrt[3]{(343)^{-2}}$

OR

If $\sqrt{7} = 2.646$ then find $\frac{1}{\sqrt{7}}$.

8. Draw the graph of $y - 4x = 8$.

9. Find the value of k, if $x = 1$, $y = 1$ is a solution of the equation $9kx + 12ky = 63$.

10. A right triangle with its sides 3 cm, 4 cm and 5 cm is rotated about its side of 4 cm to form a cone having base radius as 3 cm. Find the volume of the solid so generated. ($\pi = 3.14$)

11. How many litres of water flow out through a pipe having 5 cm^2 area of cross section in one minute, if the speed of water in the pipe is 30 cm/sec?

OR

Find the volume and surface area of a sphere of radius 21 cm.

12. Factorise: $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

Section C

(Questions 13 to 22 carry 3 marks each)

13. Simplify:

$$\frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

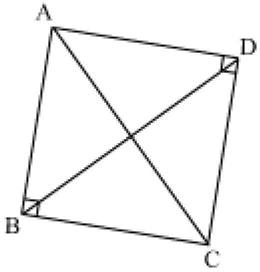
14. Which of the following expressions are polynomial in one variable? State reasons for your answers:

(i) $\frac{(x+1)(x+2)}{x}$ (ii) $t^2(t^2-3)$

(iii) $\frac{1}{2}(x^2+4x+5)$ (iv) $\sqrt{3}x^2+6\sqrt{x}$

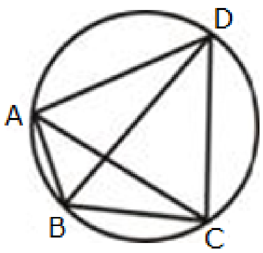
(v) $z+\frac{1}{z}$

15. ΔABC and ΔADC are two right triangles with common hypotenuse AC. Prove that ABCD is a cyclic quadrilateral and hence prove that ∠CAD = ∠CBD.



OR

In the given figure, ABCD is a cyclic quadrilateral, in which AC and BD are the diagonals. If m ∠DBC = 55° and m ∠BAC = 45°, find m ∠BCD.



16. Factorize: $(x - 3y)^3 + (3y - 7z)^3 + (7z - x)^3$.

17. Draw a line segment of length 8 cm and bisect it.

18. A bag contains 12 balls out of which x are white. If one ball is taken out from the bag, find the probability of getting a white ball. If 6 more white balls are added to the bag and the probability now for getting a white ball is double the previous one, find the value of x.

OR

The numbers 50, 42, 35, (2x + 10), (2x – 8), 12, 11, 8 have been written in a descending order. If their median is 25, find the value of x.

19. Draw the graph of 2x + 3y = 11. From graph, find the value of x, if y = 5.

20. The polynomials $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 and R_2. Find 'a' if $R_1 + R_2 = 0$.

21. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

OR

A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius of the tank is 1 m then find the volume of iron used in the tank.

22. Fifty seeds each were selected at random from 5 bags of seeds, and were kept under standardized conditions favorable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Bags	1	2	3	4	5
Number of germinated seeds	40	48	42	39	41

What is the probability of

i. More than 40 seeds germinating in a bag?
ii. 49 seeds germinating in a bag?
iii. More than 35 seeds germinating in a bag?

OR

The mean of 25 observations is 36. Out of these observations, if the mean of first 13 observations is 32 and that of the last 13 observations is 40, find the 13th observation.

Section D
(Questions 23 to 30 carry 4 marks each)

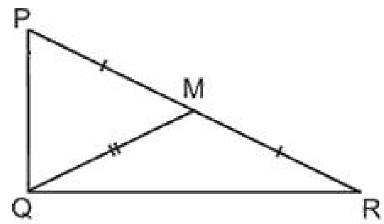
23. Find the value of:

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

24. Construct ΔXYZ in which $m\angle Y = 30^\circ$, $m\angle Z = 90^\circ$ and $XY + YZ + ZX = 11$ cm.

25. Simplify: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

26. If M is the mid-point of the hypotenuse PR of a right-angled triangle PQR, prove that $QM = \frac{1}{2}PR$

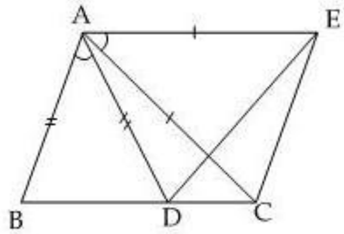


27. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm. The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

OR

How many cubic meters of earth must be dug out to sink a well 14 m deep and having a radius of 4 m? If the earth taken out is spread over a plot of dimensions (25 m × 16 m), what is the height of the platform so formed?

28. In t he figure, AC = AE, AB = AD and ∠BAD = ∠EAC show that BC = DE.



29. How does Euclid's fifth postulate imply the existence of parallel lines? Give a mathematical proof.

OR

Write which of the following statements are true and which are false.

1. Euclid's fourth axiom says that everything equals itself.
2. The Euclidean geometry is valid only for figures in the plane.
3. Part of a line with two end points is called a line segment.
4. A simple closed figure made up of three or more line segments is called a polygon.

30. The polynomials $x^3 + 2x^2 - 5ax - 8$ and $x^3 + ax^2 - 12x - 6$ when divided by (x – 2) and (x – 3) leave remainders p and q, respectively. If q – p = 10, then find the value of a.

OR

Factorise : $x^6 - 7x^3 - 8$

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Class IX Mathematics
Sample Paper 7 – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1.

$$\left(\sqrt{5}+\sqrt{6}\right)^2 = 5+6+2\sqrt{30} \qquad .....\because (a+b)^2 = a^2+2ab+b^2$$

$$= 11+2\sqrt{30}$$

On compairing $a + b\sqrt{30}$ and $11 + 2\sqrt{30}$, we get

a = 11 and b = 2

OR

$$\left(\sqrt{3}+2\right)^2 = \left(\sqrt{3}\right)^2 + 2\times\sqrt{3}\times 2 + 2^2$$

$$= 3+4\sqrt{3}+4$$

$$= 7+4\sqrt{3}$$

Hence, $\left(\sqrt{3}+2\right)^2$ it is irrational number.

2. At x = –1,

$f(x) = 8(-1)^2 - 3(-1) + 7 = 8 + 3 + 7 = 18$

∴ The value of f(x) at x = –1 is 18.

OR

$x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 3^2) = (x + 3)(x^2 - 3x + 9)$

3. We know that a quadrilateral with two separate pairs of equal adjacent sides is called a kite.

∴ □PQRS is a Kite.

4. The given equation is 3x = 4.

$\therefore x = \dfrac{4}{3}$

As the Graph of equation x = k (k is any constant) is parallel to the y-axis.

∴ The graph of the linear equation 3x = 4 is parallel to y-axis.

5. Here, class size = 15, maximum value = 159 and minimum value = 69.

$\therefore$ Range = 159 – 69 = 90

Number of classes = $\frac{\text{range}}{\text{class size}} = \frac{90}{15} = 6$

Therefore, the number of classes is 6.

6. In triangle, a = 6 cm, b = 8 cm, c = 10 cm

$\therefore$ semi-perimeter $= s = \frac{a + b + c}{2} = \frac{6+8+10}{2} = 12$ cm

Section B

7. $\sqrt[3]{(343)^{-2}} = (343)^{-2/3}$

$= [(7)^3\}^{-2/3}$

$= 7^{3 \times -2/3}$

$= 7^{-2}$

$= \frac{1}{49}$

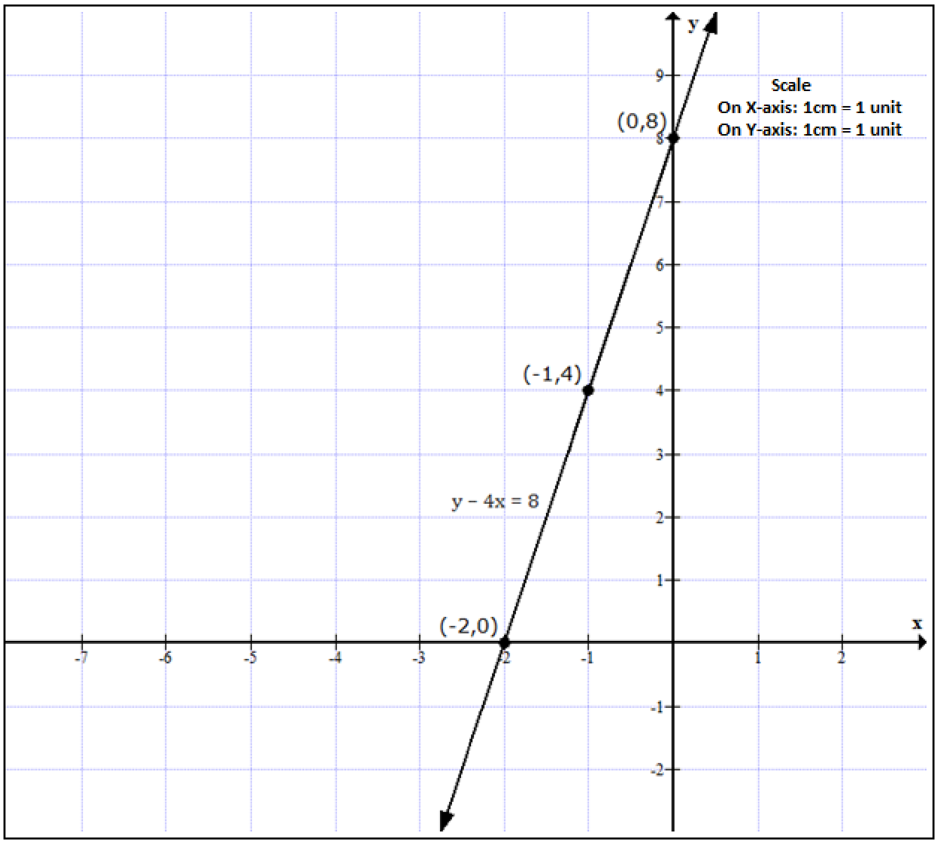
OR

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7} = \frac{2.646}{7} = 0.378$$

8. The given equation is y – 4x = 8 ⇒ y = 8 + 4x

x	0	–1	–2
y	8	4	0

Plot the points (0, 8), (–1, 4) and (–2, 0). Draw a line passing through these points.



9. Since x = 1, y = 1 is the solution of 9kx + 12ky = 1, it will satisfy the equation.

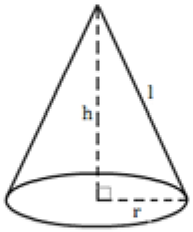
$\therefore$ 9k(1) + 12k(1) = 63

$\therefore$ 9k + 12k = 63

$\therefore$ 21k = 63

$\therefore$ k = 3

10. When rotated about the side of 4 cm.



Given,

r = 3 cm, h = 4 cm, l = 5 cm

Volume of solid = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times (3)^2 \times 4\right) cm^3 = 37.68\ cm^3$

11. Area of cross section of pipe = 5 cm^2

Speed of water flowing out of the pipe = 30 cm/sec

Volume of water that flows out in 1 sec = 5 × 30 = 150 cm^3

Volume of water that flows out in 1 minute = 150 × 60 = 9000 cm^3 = 9 litres.

OR

Radius of a sphere = 21 cm

Volume of a sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 21\times 21\times 21 = 38808\ cm^3$

Surface area = $4\pi r^2 = 4\times\frac{22}{7}\times 21\times 21 = 5544\ cm^3$

12.

$$x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$$
$$=\left(x^2+\frac{1}{x^2}+2\right)-2\left(x+\frac{1}{x}\right)$$
$$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)$$
$$=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$$

Section C

13. $$\frac{(25)^{\frac{3}{2}}\times(343)^{\frac{3}{5}}}{16^{\frac{5}{4}}\times 8^{\frac{4}{3}}\times 7^{\frac{3}{5}}}$$
$$=\frac{\left(5^2\right)^{\frac{3}{2}}\times\left(7^3\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}}\times\left(2^3\right)^{\frac{4}{3}}\times 7^{\frac{3}{5}}}$$
$$=\frac{5^3\times 7^{\frac{9}{5}}}{2^5\times 2^4\times 7^{\frac{3}{5}}}$$
$$=\frac{5^3\times 7^{\frac{9}{5}}}{2^9\times 7^{\frac{3}{5}}}$$
$$=\frac{5^3\times 7^{\frac{6}{5}}}{2^9}$$

14.

i. No.

$\frac{x^2+3x+2}{x}=x+3+2x^{-1}$ has a negative power of x.

ii. Yes

$t^2\left(t^2-3\right)$

t^4-3t^2

iii. Yes

$\frac{\left(x^2+4x+5\right)}{2}=\frac{x^2}{2}+\frac{4x}{2}+\frac{5}{2}=\frac{x^2}{2}+2x+\frac{5}{2}$

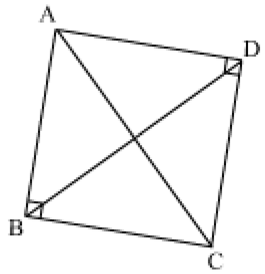
iv. No

$\sqrt{3}x^2+6\sqrt{x}=\sqrt{3}x^2+6\left(x\right)^{1/2}$ has fractional power of x.

v. No

$z+\frac{1}{z}$ i.e. $z+z^{-1}$ has a negative power of x.

15.



In ΔABC

$m\angle ABC + m\angle BCA + m\angle CAB = 180°$ (Angle sum property of a triangle)

$\Rightarrow 90° + m\angle BCA + m\angle CAB = 180°$

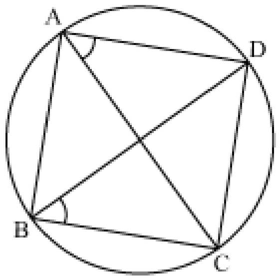
$\Rightarrow m\angle BCA + m\angle CAB = 90°$ ... (1)

In ΔADC

$m\angle CDA + m\angle ACD + m\angle DAC = 180°$ (Angle sum property of a triangle)

$\Rightarrow 90° + m\angle ACD + m\angle DAC = 180°$

$\Rightarrow$ m$\angle$ACD + m$\angle$DAC = 90° ... (2)



Adding equations (1) and (2), we have

m$\angle$BCA + m$\angle$CAB + m$\angle$ACD + m$\angle$DAC = 180°

$\Rightarrow$ (m$\angle$BCA + m$\angle$ACD) + (m$\angle$CAB + m$\angle$DAC) = 180°

m$\angle$BCD + m$\angle$DAB = 180° ... (3)

But it is given that

m$\angle$B + m$\angle$D = 90° + 90° = 180° ... (4)

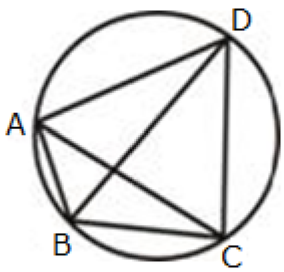
From equations (3) and (4), we can see that quadrilateral ABCD is having sum of measures of opposite angles as 180°.

So, it is a cyclic quadrilateral.

Consider chord CD.

Now, $\angle$CAD = $\angle$CBD (Angles in same segment)

OR



Given: ABCD is a cyclic quadrilateral in which AC and BD are diagonals.
m $\angle$DBC = 55° and m $\angle$BAC = 45°
To find: m $\angle$BCD

Proof: m $\angle$CAD = m $\angle$DBC = 55° (Angles in the same segment)

Therefore, m $\angle$DAB = m $\angle$CAD + m $\angle$BAC

= 55° + 45°

= 100°

However, m $\angle$DAB + m $\angle$BCD = 180°

(∵ opposite angles of a cyclic quadrilateral)

So, m ∠BCD = 180° – 100° = 80°

16. We know that if a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

Here,

$x - 3y + 3y - 7z + 7z - x = 0$

i.e. $a + b + c = 0$

$\therefore\ (x-3y)^3 + (3y-7z)^3 + (7z-x)^3$

$= 3(x-3y)(3y-7z)(7z-x)$

So factors are 3, $(x-3y), (3y-7z), (7z-x)$.

17. Steps of construction:

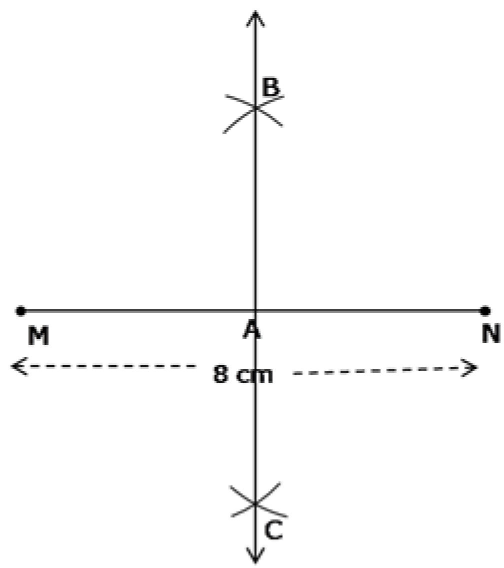
i. Draw a line segment MN = 8 cm

ii. Taking M as the centre and radius more than half the length MN, draw two arcs in the upper and lower portion of MN.

iii. Taking N as the centre and the same radius, draw two arcs which cut the previous arcs at B and C.

iv. Join BC which cuts MN at A.

BC is the required perpendicular bisector of MN.



18. Number of white balls = x

Total no. of balls $=12$

$P(\text{white ball})=\dfrac{x}{12}$

If 6 white balls are added,

Total no. of balls $=18$

White ball $=x+6$

$\therefore P(\text{white ball})=\dfrac{x+6}{18}$

If 6 more white balls are added to the bag then probability for getting a white ball is doubled

$\therefore \dfrac{x+6}{18}=\dfrac{2x}{12}$

$\Rightarrow 6x+36=18x$

$\Rightarrow x=3$

OR

The numbers of observations are 8.

Hence, median is $\dfrac{\left(\frac{n}{2}\right)^{th}+\left(\frac{n}{2}+1\right)^{th}}{2}=\dfrac{\left(\frac{8}{2}\right)^{th}+\left(\frac{8}{2}+1\right)^{th}}{2}=\left(\dfrac{4^{th}+5^{th}}{2}\right)^{th}$ value

Median = $\dfrac{2x+10+2x-8}{2}=25$

4x + 2 = 50

4x = 48

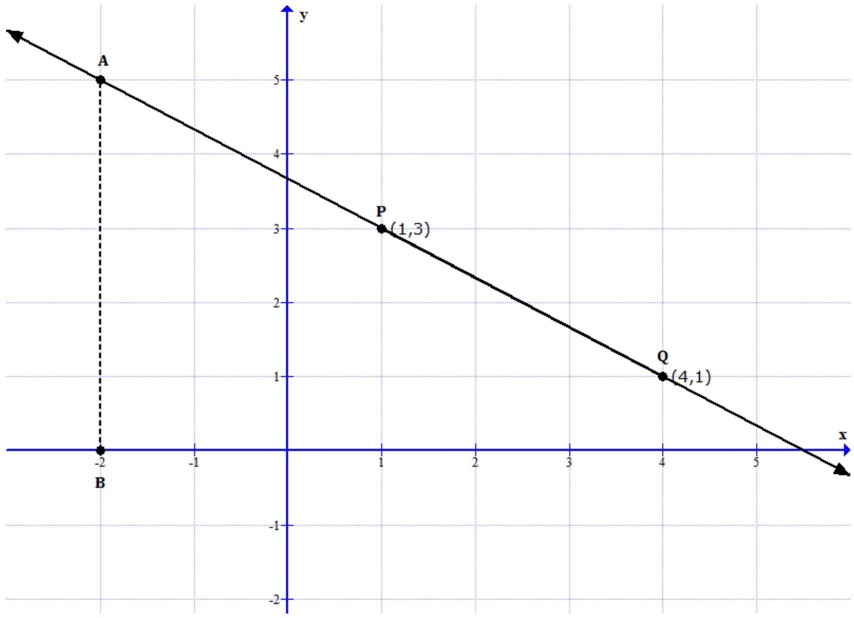
x = 12

19. The given equation is 2x + 3y = 11.

$\Rightarrow y = \dfrac{11-2x}{3}$

x	1	4
y	3	1

Plot points (1, 3), (4, 1). Draw line passing through the points.



From the graph we can see that if y = 5 then the value of x is −2.

20. When $p(x) = ax^3 + 3x^2 - 3$ is divided by $(x - 4)$, the remainder is given by

$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

When $q(x) = 2x^3 - 5x + a$ is divided by $(x - 4)$, the remainder is given by

$R_2 = 2(4)^3 - 5(4) + a = 108 + a$

Given: $R_1 + R_2 = 0$

$\Rightarrow 65a + 153 = 0 \Rightarrow a = \dfrac{-153}{65}$

By hit and trial we find x = 3 is factor of given polynomial, as

$2(3)^3 - 9 - 39 - 6 = 54 - 54 = 0$

By dividing $2x^3 - x^2 - 13x - 6$ by $x - 3$ we get $(2x^2 + 5x + 2)$ as quotient.

Factorising this further

$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2$

$= 2x(x + 2) + 1(x + 2)$

$= (2x + 1)(x + 2)$

So, $2x^3 - x^2 - 13x - 6 = (2x + 1)(x + 2)(x - 3)$

21. The given tank is cuboidal in shape having its length (l) as 20 m, breadth (b) as 15 m and height (h) as 6m.

Capacity of tank = l × b × h

⇒ Capacity of tank = (20 × 15 × 6) m^3 = 1800 m^3

⇒ Capacity of tank = 1800000 litres

Water consumed by people of the village in 1 day = 4000 × 150 litres = 600000 litres

Let us assume that the water in the tank lasts for n days.

Water consumed by all people of the village in n days = capacity of tank

⇒ n × 600000 = 1800000

⇒ n = 3

Water in this tank will last for 3 days.

OR

Inner radius of the tank = 1 m = 100 cm

Outer radius of the tank = 100 + 1 = 101 cm

Volume of iron used in the hemispherical tank

$$=\frac{2}{3}\pi\left(101^3-100^3\right)=\frac{2}{3}\times\frac{22}{7}(1030301-1000000)=63487.81\text{ cm}^3$$

Hence, the volume of iron used in the tank is 63487.81 cm^3.

22. Total number of bags is 5.

i. Number of bags in which more than 40 seeds germinated out of 50 seeds is 3.

P(germination of more than 40 seeds in a bag) = $\frac{3}{5}$ = 0.6

ii. Number of bags in which 49 seeds germinated = 0

P(germination of 49 seeds in a bag) = $\frac{0}{5}$ = 0

iii. Number of bags in which more than 35 seeds germinated = 5.

The required probability = $\frac{5}{5}=1$

OR

Mean of 25 observations = 36

Sum of 25 observations = 36 × 25 = 900

Mean of first 13 observations = 32

Sum of 13 observations = 32 × 13 = 416

Mean of last 13 observation = 40

Sum of last 13 observations = 40 × 13 = 520

13th observation = 416 + 520 − 900 = 36

Hence, 13th observation is 36.

Section D

23.

$$\frac{1}{3-\sqrt{8}}=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{8-7}=\sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}-\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}-\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=5$$

24. The steps of construction for the required triangle are as follows:

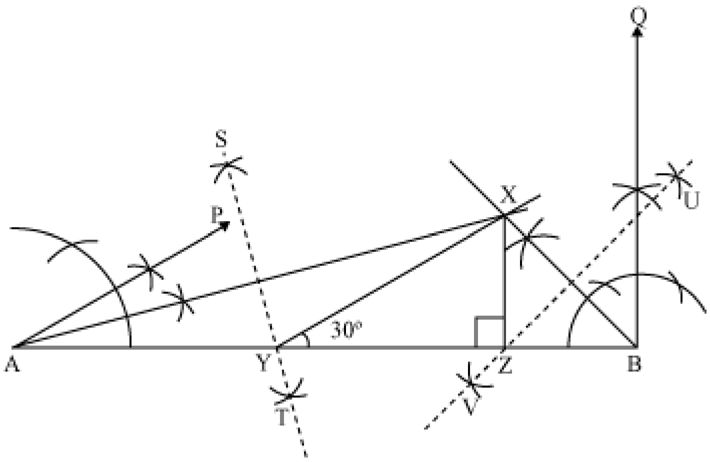
Step I: Draw a line segment AB of 11 cm (As XY + YZ + ZX = 11 cm).

Step II: Construct ∠PAB of 30° at point A and an angle ∠QBA of 90° at point B.

Step III: Bisect ∠PAB and ∠QBA. Let these bisectors intersect each other at point X.

Step IV: Draw perpendicular bisector ST of AX and UV of BX.

Step V: Let ST intersects AB at Y and UV intersects AB at Z. Join XY and XZ. ΔXYZ is the required triangle.



25. Consider $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

We know that,

If x + y + x = 0 then $x^3 + y^3 + z^3 = 3xyz$

Now, $a^2 - b^2 + b^2 - c^2 - a^2 = 0$

And, a – b + b – c + c – a = 0

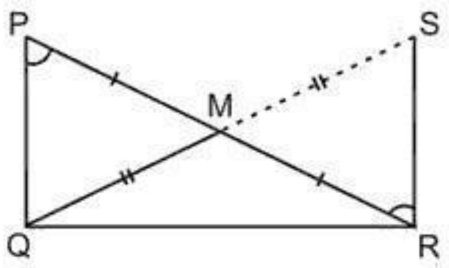
$$\therefore \frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$=\frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$$

$$=\frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

$$=(a+b)(b+c)(c+a)$$

26.



Produce QM to S such that QM = MS. Join SR

In ΔPMQ and ΔRMS

PM = MR (M is the mid point)

QM = MS (By construction)

$\angle PMQ = \angle RMS$...vertically opposite angles

$\therefore \Delta PMQ \cong \Delta RMS$ (SAS congruence criterion)

$\therefore PQ = SR$ and $\angle QPM = \angle SRM$ (c.p.c.t)

$\angle QPM = \angle SRM$ (alternate angles) $\therefore RS \parallel PQ$

$\angle PQR + \angle QRS = 180^\circ$ (Co–interior angles)

$\Rightarrow 90^\circ + \angle QRS = 180^\circ$

$\Rightarrow \angle QRS = 90^\circ$

In ΔPQR and ΔQRS

QR = RQ (common)

PQ = SR

$\angle PQR = \angle QRS$ (90° each)

$\therefore \Delta PQR \cong \Delta SRQ$ (SAS congruence criterion)

$\therefore PR = QS \Rightarrow \frac{1}{2}SQ = \frac{1}{2}PR$

$\Rightarrow \frac{1}{2}SQ = QM = \frac{1}{2}PR$

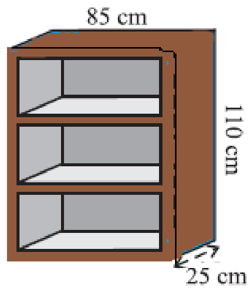
Hence, proved

27. Given,

External length (l) of bookshelf = 85 cm.

External breadth (b) of bookshelf = 25 cm.

External height (h) of bookshelf = 110 cm.



External surface area of shelf excluding its front face

$= lh + 2(lb + bh)$

External surface area of shelf excluding its front face

$= [85 \times 110 + 2(85 \times 25 + 25 \times 110)]\ cm^2$

External surface area of shelf excluding its front face = 19100 cm^2

Area of front face = $[85 \times 110 - 75 \times 100 + 2(75 \times 5)]\ cm^2$

$= 1850 + 750\ cm^2 \quad = 2600\ cm^2$

Area to be polished = $(19100 + 2600)\ cm^2 = 21700\ cm^2$

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area = Rs. (21700×0.20) = Rs. 4340

Now, length (l), breadth (b) height (h) of each row of bookshelf is 75 cm, 20 cm, and 30 cm respectively.

Area to be painted in 1 row = $2(l + h)b + lh$

$= [2(75 + 30) \times 20 + 75 \times 30]\ cm^2$

$= (4200 + 2250)\ cm^2$

$= 6450\ cm^2$

Area to be painted in 3 rows = $(3 \times 6450)\ cm^2 = 19350\ cm^2$

Cost of painting 1 cm^2 area = Rs. 0.10

Cost of painting 19350 cm^2 area = Rs. (19350×0.10) = Rs. 1935

Total expenses required for polishing and painting the surface of the bookshelf

= Rs. (4340 + 1935) = Rs. 6275

OR

We have r = 4 m and h = 14 m

Volume of the earth dug out of the well = $\pi r^2 h = \frac{22}{7} \times 4 \times 4 \times 14 = 704\ m^3$

Area of the given plot = 25 × 16 = 400 m^2

Volume of the platform formed = Volume of the earth dug out = 704 m^2

Height of the platform = $\frac{\text{volume in } m^3}{\text{area in } m^2} = \frac{704}{400} = \frac{176}{100} = 1.76\ m$

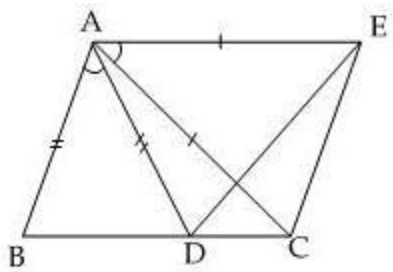
28. Given: AC = AE, AB = AD and ∠BAD = ∠EAC

To prove: BC = DE

Proof: ∠BAD = ∠EAC (given)

⇒∠BAD + ∠DAC = ∠EAC + ∠DAC

⇒∠BAC = ∠DAE



Now in ΔABC and ΔADE

AB = AD

∠BAC = ∠DAE

AC = AE

Thus, ΔABC ≅ ΔADE

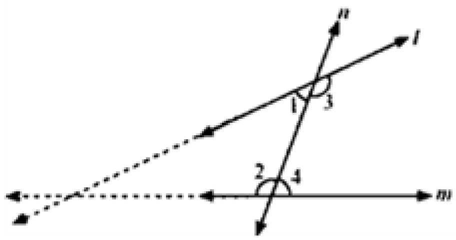
⇒ BC = DE

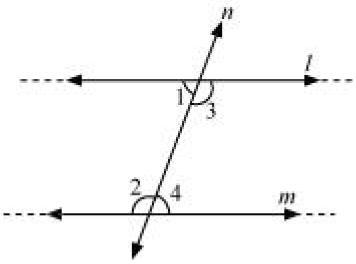
29. Euclid's 5th postulate states that:

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

This implies that if n intersects lines l and m and if ∠1 + ∠2 < 180^0, then ∠3 + ∠4 > 180^0. In that case, producing line l and further will meet in the side of ∠1 and ∠2 which is less than 180^o.

If ∠1 + ∠2 = 180^0, then ∠3 + ∠4 = 180^0





In that case, the lines l and m neither meet at the side of $\angle 1$ and $\angle 2$ nor at the side of $\angle 3$ and $\angle 4$ implying that the lines l and m will never intersect each other. Therefore, the lines are parallel.

OR

1. True
2. True, as it fails on the curved surfaces.
3. True
4. True

30. Let $f(x) = x^3 + 2x^2 - 5ax - 8$ and $g(x) = x^3 + ax^2 - 12x - 6$

When divided by $(x - 2)$ and $(x - 3)$, $f(x)$ and $g(x)$ leave remainder p and q respectively

$f(x) = x^3 + 2x^2 - 5ax - 8$

$\therefore f(2) = 2^3 + 2 \times 2^2 - 5a \times 2 - 8$

$= 8 + 8 - 10a - 8$

$\therefore p = 8 - 10a$ ....(1)

$g(x) = x^3 + ax^2 - 12x - 6$

$\therefore g(3) = 3^3 + a \times 3^2 - 12 \times 3 - 6$

$= 27 + 9a - 36 - 6$

$\therefore q = -15 + 9a$ ....(2)

If $q - p = 10$

$\Rightarrow -15 + 9a - 8 + 10a = 10$

$\Rightarrow 19a - 23 = 10$

$\Rightarrow 19a = 33$

$\therefore a = \frac{33}{19}$

OR

$x^6 - 7x^3 - 8$

Put $x^3 = y$

$x^6 - 7x^3 - 8$

$x^6 - 7x^3 - 8 = y^2 - 7y - 8$

$= y^2 - 8y + y - 8$

$= y(y - 8) + (y - 8)$

$= (y - 8)(y + 1)$

$= (x^3 - 8)(x^3 + 1)$ $\quad \because x^3 = y$

$= (x^3 - 2^3)(x^3 + 1)$

$= (x - 2)(x^2 + 2x + 4)(x + 1)(x^2 - x + 1)$

$= (x - 2)(x + 1)(x^2 + 2x + 4)(x^2 - x + 1)$

$\therefore x^6 - 7x^3 - 8 = (x - 2)(x + 1)(x^2 + 2x + 4)(x^2 - x + 1)$